Hypothesis testing when a nuisance parameter is present only under the alternative - revisited

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The Higgs Boson

- "Observation of a New Particle in the Search for the Standard Model Higgs Boson with the ATLAS Detector at the LHC"
- *The global significance of a local 5.9σ excess anywhere in the mass range 110–600GeV is estimated to be approximately 5.1σ, increasing to 5.3σ in the range 110–150GeV*.
- See http://arxiv.org/pdf/1207.7214v2

The sigma values

- The relationship between these σ values is derived from my papers on "Hypothesis testing when a nuisance parameter is present only under the alternative".

What I am going to talk about

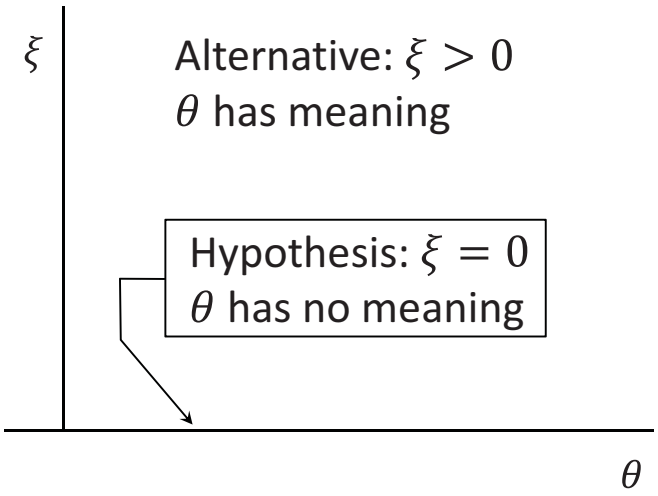
- the results in my papers;
- how they apply to the Higgs Boson data;
- the original motivation for the papers;
- other applications;
- what else one could look at.

Trilogy of papers

- (1977) Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* **64** 247-254.
- (1987) Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* **74** 33-43.
- (2002) Hypothesis testing when a nuisance parameter is present only under the alternative - linear model case. *Biometrika* **89** 484-489.
- See http://robertnz.net for scanned copies.

Suppose

- The distribution of the random variables representing the outcome of an experiment depends on two parameters ξ and θ.
- We wish to test the hypothesis $\xi = 0$ against the alternative $\xi > 0$.
- The distribution does not depend on θ when $\xi = 0$.



Then

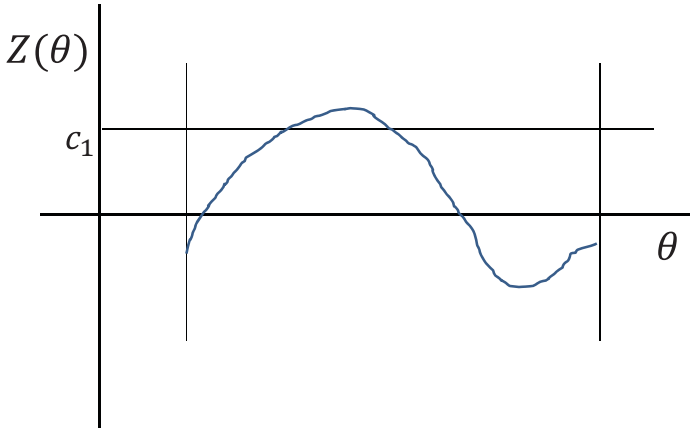
- Standard asymptotic methods such as likelihood ratio testing or $C(\alpha)$ testing are not directly applicable.
- However, these methods may, under appropriate conditions, be used to reduce the problem to one involving inference from a Gaussian process.

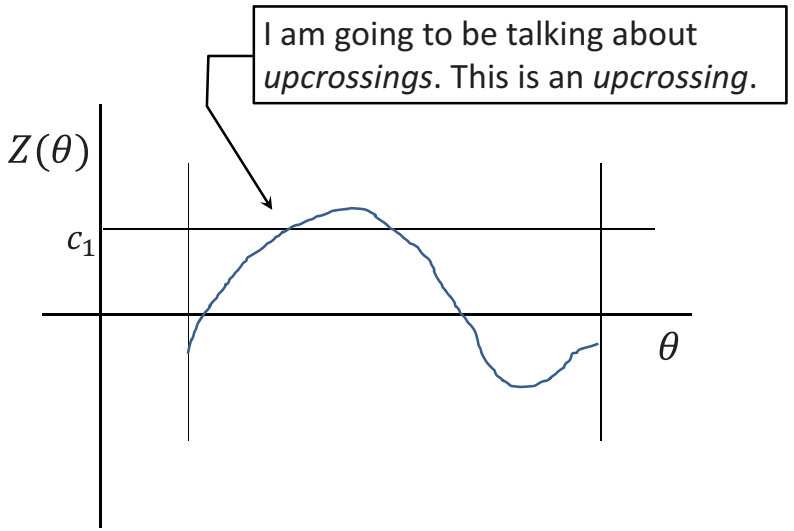
Suppose

- If θ was known we could find an asymptotically optimal test for testing $\xi = 0$ against $\xi > 0$.
- This test has critical region $\{Z(\theta) > c\}$
- Asymptotically, $Z(\theta)$ is $N(0,1)$ under the hypothesis.
- Asymptotically, $Z(\theta)$ has positive mean under the alternative.

Then

- I suggest using a test with critical region of the form
$$\sup\{Z(\theta)\} > c_1$$
where θ ranges over its possible values.
- Note: In all my results θ is one dimensional and has a bounded range of values.





How to calculate significance level

- If $\sup\{Z(\theta)\} > c_1$ then $Z(\theta)$ must upcross c_1 (count $Z(\theta)$ starting above c_1 as an upcrossing).
- $$\Pr\{\sup\{Z(\theta)\} > c_1\} = \Pr(\text{at least one upcrossing}) \leq E(\text{number of upcrossings}).$$
- If $\{Z(\theta)\}$ can be considered as a Gaussian process and is differentiable then there is a formula for this ("The Rice formula").
- I think I learnt about this from either Cramer or Leadbetter.

My papers

- The first looks at the problem I have just described and its two-sided version and finds the upper bound on the significance probability.
- In the second ξ is a vector, we are testing $\xi = 0$ against $\xi \neq 0$ and the test statistic for known θ is χ^2. There is also a quick approximate method for calculating the significance probability.
- In the third there is an unknown variance so the distributions are either *t* or *F*.

Notes

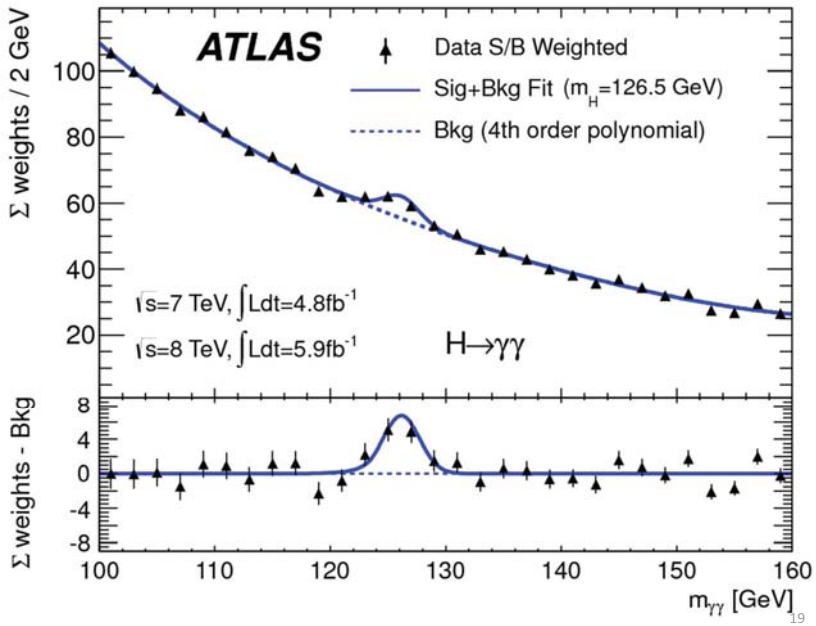
- The first two of these papers are my best known papers.
- Each of the first two should reach 1000 citations in *Google Scholar* sometime next year.
- Both of the first two were rejected on their first submission.

Roger

- Roger Littlejohn helped me with the first paper and should probably have been a co-author.

The Higgs Boson

- The Higgs boson detection experiment involves measuring the rate at which "interesting" interactions occur over a range of energies and searching for a peak.
- There were two semi-independent sets of experiments – I am using the results for the "Atlas" team as they had graphs that I could present better.
- The next slide has their most important graph.



The ATLAS graph

- The top graph gives the occurrence rates at which the reaction the Higgs boson is expected to facilitate over a range of energies.
- If the Higgs boson exists there should be a peak above background. Theory says how big and wide the peak should be, but not where it should be.
- The bottom graph gives the difference between the top graph and the fitted background.

The Atlas graph

- In my notation ξ denotes the size of the peak above background and θ denotes its location.
- They want to know if $\xi > 0$.
- If $\xi = 0$ then there is no Higgs Boson and θ is meaningless.

P-values and σ values

- Rather than give a significance probability (p-value) like 2.5% the physicists give the corresponding deviation of a normal random variable, 2σ, in this case.
- 5.9σ corresponds to a p-value of 2×10^{-9}.
- 5.3σ corresponds to a p-value of 6×10^{-8}.
- 5.1σ corresponds to a p-value of 2×10^{-7}.

The Higgs Boson p-values

- *The global significance of a local 5.9σ excess anywhere in the mass range 110–600GeV is estimated to be approximately 5.1σ, increasing to 5.3σ in the range 110–150GeV*.
- The *5.9σ* value is when they are just looking at the maximum of the $Z(\theta)$ graph.
- They are not sure what range of energies they should be scanning over so they give the reduced values for two ranges.

The Higgs Boson p-values

- The calculation is based on a combination of my formula and simulation.
- See *Procedure for the LHC Higgs boson search combination in Summer 2011* http://cdsweb.cern.ch/record/1379837/files/NOTE2011_005.pdf
- Questions
  - Does the non-Gaussian nature of the data matter?
  - Is their method a good approach?
  - They seem to have stopped shortly after they got their 5σ significance – does that matter?

How it started

- The temperatures of a number of codling moth pupa were measured over winter. Development was supposed to proceed at a rate proportional to temperature above some critical temperature with no development below that temperature.
- Try to verify this, given the times of emergence.
- The critical temperature was a parameter that was present only under the alternative.
- That analysis went nowhere but it got me thinking about problem.

Other examples

- Mixture of normals (Jin Seo Cho talked about this at previous conference)
- Mixture of exponentials
- Continuous change point
- Jump change point (upcrossing formula doesn't work here)
- Peaks in spectral analysis
- Linkage in genetics
- Hidden Markov process

Projects

- What about two-dimensional θ?
- Check asymptotics for Higgs Boson analysis
- Is there a sensible analogue for AIC?
- Is bootstrap or jack-knife applicable?